Sample Question Paper - 4
Mathematics-Basic (241)
Class- X, Session: 2021-22
TERM II

Time Allowed: 2 hours **Maximum Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
3. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
4. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study-based questions.

Section A

1. Find whether the equation has real roots. If real roots exist, find them: $5x^2 - 2x - 10 = 0$ **[2]**

OR

Find the nature of the roots of the following quadratic equation. If the real roots exist, find them: $3x^2 - 4\sqrt{3}x + 4 = 0$

2. The radii of the bases of a cylinder and a cone are in the ratio 3 : 4 and their heights are in the ratio 2 : 3. What is the ratio of their volumes? **[2]**

3. Calculate the mean of the following data, using direct method: **[2]**

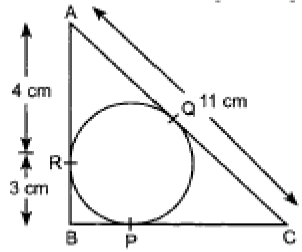
Class	25 - 35	35 - 45	45 - 55	55 - 65	65 - 75
Frequency	6	10	8	12	4

4. Is 12, 2 , - 8 , -18 ,.... an arithmetic progression. If yes, find out the common difference. **[2]**

5. Find the mode of the following frequency distribution: **[2]**

Class	0 - 20	20 - 40	40 - 60	60 - 80	80 -100
Frequency	25	16	28	20	5

6. In figure, $\triangle$ABC is circumscribing a circle. Find the length of BC. **[2]**



OR

Let A be the one of the points of intersection of two intersecting circles with centres O and Q. The tangents at A to the two circles meet the circles again at B and C, respectively. Let the point P be located so that OPAQ is a parallelogram. Prove that P is the circumcentre of the triangle ABC.

Section B

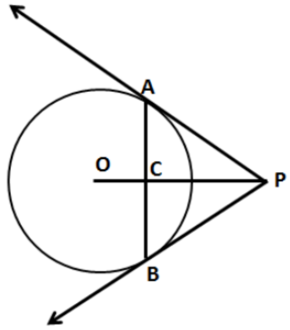
7. If $a_n = 3 - 4n$, show that a_1, a_2, a_3... form an A.P. Also find S_{20}. **[3]**

8. The shadow of a tower standing on a level ground is found to be 40 m longer when the Sun's altitude is 30^o, than when it is 60^o. Find the height of the tower. **[3]**

OR

The pilot of an aircraft flying horizontally at a speed of 1200 km/hr. observes that the angle of depression of a point on the ground changes from 30° to 45° in 15 seconds. Find the height at which the aircraft is flying.

9. From a point P outside a circle with centre O, tangents PA and PB are drawn to the circle. Prove that OP is the right bisector of the line segment AB. **[3]**



10. Find the values of p for which the quadratic equation $(2p + 1)x^2 - (7p + 2)x + (7p - 3) = 0$ has real and equal roots. **[3]**

Section C

11. Draw two concentric circles of radii 4 cm and 6 cm. Construct a tangent to the smaller circle from a point on the larger circle. Measure the length of this tangent. **[4]**

OR

Construct tangents to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm.

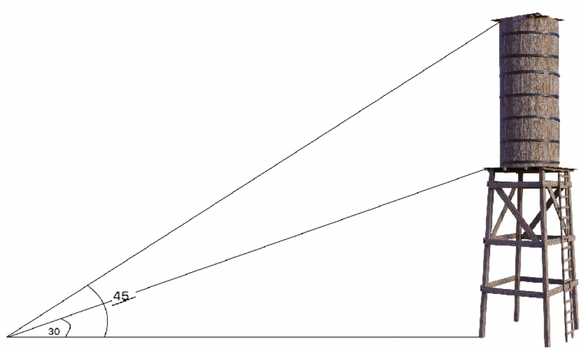
12. Calculate the mode of the following frequency distribution table : **[4]**

Marks	Number of students
25 or more than 25	52
35 or more than 35	47
45 or more than 45	37
55 or more than 55	17
65 or more than 65	8
75 or more than 75	2
85 or more than 85	0

13. In a society, there are many multistorey buildings. The RWA of the society wants to install a tower and a watertank so that all the households can get water without using water pumps. For this they have measured the height of the tallest building in their society and now they want to install a tower that will be taller than that so that the level of water must be higher **[4]**

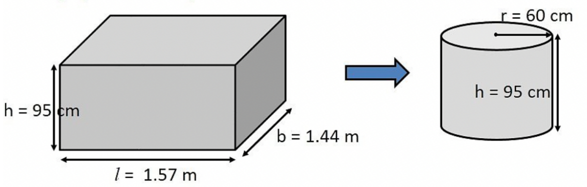
than the tallest building in their society. Here is one solution they have found and now they want to check if it will work or not.

From a point on the ground 40 m away from the foot of a tower, the angle of elevation of the top of the tower is 30^0. the angle of elevation of the top of a water tank (on the top of the tower) is 45^0.



i. Find the height of the tower.
ii. Find the depth of the tank.

14. Selvi's house has an overhead tank in the shape of a cylinder. This is filled by pumping water from a sump (underground tank). Which is in the shape of a cuboid. The sump has dimensions $1.57\text{m} \times 1.44\text{m} \times 0.95\text{m}$. The overhead tank has its radius of 60 cm and its height is 95 cm. **[4]**



i. Find the height of the water, left in the sump after the overhead tank has been completely filled with water from a sump which had been full.
ii. Compare the capacity of the tank with that of the sump. (Use $\pi = 3.14$).

Solution

MATHEMATICS BASIC 241

Class 10 - Mathematics

Section A

1. To check whether the quadratic equation has real roots or not, we need to check the discriminant value i.e.,

   D = b^2 - 4ac

   Given, $5x^2 - 2x - 10 = 0$

   $\therefore$ D = $(-2)^2 - 4(5)(-10)$

   $\Rightarrow$ D = 4 + 200 > 0

   Hence, the roots are real and distinct.

   To find the roots, use the formula,

   $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

   $\therefore x = \frac{2 \pm \sqrt{204}}{2(5)}$

   $= \frac{2 \pm 2\sqrt{51}}{10} = \frac{1+\sqrt{51}}{5}, \frac{1-\sqrt{51}}{5}$

   OR

   $3x^2 - 4\sqrt{3}x + 4 = 0$

   Comparing this equation with general equation $ax^2 + bx + c = 0$,

   We get a = 3, b = $-4\sqrt{3}$ and c = 4

   Discriminant = b^2 - 4ac = $(-4\sqrt{3})^2 - 4(3)(4)$ = 48 - 48 = 0

   Discriminant is equal to zero which means equations has equal real roots.

   Applying quadratic $x = \frac{-b \pm \sqrt{b^2 - 4a}}{2a}$ to find roots,

   $x = \frac{4\sqrt{3} \pm \sqrt{0}}{6} = \frac{2\sqrt{3}}{3}$

   Because, equation has two equal roots, it means $x = \frac{2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3}$

2. Radii of the bases of a cylinder and a cone = 3 : 4

   and ratio in their heights = 2 : 3

   Let r_1, r_2 be the radii and h_1 and h_2 be their heights.

   Then, $\frac{r_1}{r_2} = \frac{3}{4}$ and $\frac{h_1}{h_2} = \frac{2}{3}$

   Now $\frac{\text{Volume of cylinder}}{\text{Volume of cone}}$

   $= \frac{\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2}$

   $= \frac{3r_1^2 h_1}{r_2^2 h_2} = 3\left(\frac{r_1}{r_2}\right)^2 \times \left(\frac{h_1}{h_2}\right)$

   $= 3 \times \left(\frac{3}{4}\right)^2 \times \frac{2}{3}$

   $= 3 \times \frac{9}{16} \times \frac{2}{3} = \frac{9}{8}$

   $\therefore$ Ratio of their volumes = 9 : 8

3.

Class Interval	Frequency	Class mark x_i	$f_i x_i$
25 - 35	6	30	180
35 - 45	10	40	400
45 - 55	8	50	400
55 - 65	12	60	720
65 - 75	4	70	280
	$\Sigma f_i = 40$		$\Sigma(f_i x_i) = 1980$

from table ,

$\Sigma f_i = 40 , \Sigma(f_i x_i) = 1980$

we know that,

mean = $\frac{\Sigma f_i x_i}{\Sigma f_i}$

$= \frac{1980}{40}$

$= 49.5$

4. Given sequence is 12, 2, -8, -18

Here,

First term (a) = 12

a_2 = 2

a_3 = -8

Now, for the given sequence, we must have

Common difference (d) = a_1 - a = a_2 - a_1

Here,

a_1 - a = 2 - 12 = - 10

a_2 - a_1 = -8 - 2 = -10

Since a_1 - a = a_2 - a_1

Hence, the given sequence is an A.P with the common difference d = -10

5.

Class	0-20	20-40	40-60	60-80	80-100
Frequency	25	16	28	20	5

Clearly, the modal class is 40-60 , as it has the maximum frequency.

∴ x_k = 40 , h = 10, f_k = 28, f_{k-1} = 16, f_{k+1} = 20

Mode, M_o = $x_k + \left\{h \times \frac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})}\right\}$

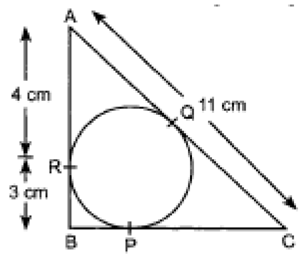
$= 40 + 10\left\{\frac{28-16}{2(28)-16-20}\right\}$

$= 40 + 10 \times \frac{12}{20}$

= 40 + 6

= 46

6. Given,



$AR = 4\ cm.$

Also, $AR = AQ \Rightarrow AQ = 4cm$

Now, $QC = AC - AQ$

$= 11cm - 4cm = 7cm$ ...(i)

Also, $BP = BR$

∴ BP = 3 cm and PC = QC

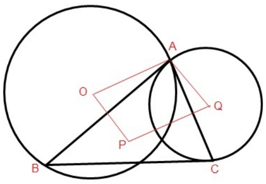
∴ PC = 7 cm [From (i)]

$BC = BP + PC$

$= 3cm + 7cm$

$= 10cm$

OR



Given that A be the one of the points of intersection of two intersecting circles with centres O and Q. The tangents at A to the two circles meet the circles again at B and C, respectively and AOPQ is a parallelogram. Now In order to prove that P is the circumcentre of ΔABC, it is sufficient to show that P is the point of intersection of perpendicular bisectors of the sides of Δ ABC i. e, OP and PQ are perpendicular bisectors of sides AB and AC respectively.

Now, AC is tangent at A to the circle with centre at O and OA is its radius.

$\therefore \quad OA \perp AC$

$\Rightarrow \quad PQ \perp AC$ [$\because OAQP$ is a parallelogram $\therefore OA \| PQ$]

=> PQ is the perpendicular bisector of AC.[$\because Q$ is the centre of the circle]

Similarly, BA is the tangent at A to the circle with centre at Q and AQ is its radius through A.

$\therefore \quad BA \perp AQ$

$\therefore \quad BA \perp OP$ [$\because AQPO$ is parallelogram $\therefore OP \parallel AQ$]

$\Rightarrow$ OP is the perpendicular bisector of AB.

Thus, P is the point of intersection of perpendicular bisectors PQ and PO of sides AC and AB respectively

Hence, P is the circumcentre of ΔABC.

Section B

7. Given that, nth term of the series is $a_n = 3 - 4n$

For a_1,

Put n = 1 so a_1 = 3 - 4(1) = - 1

For a_2,

Put n = 2, so a_2 = 3 - 4(2) = - 5

For a_3,

Put n = 3 so a_3 = 3 - 4(3) = - 9

For a_4,

Put n = 4 so a_4 = 3 - 4(4) = - 13

So AP is - 1, - 5, - 9, - 13, ...

$a_2 - a_1 = -5 - (-1) = -4$

$a_3 - a_2 = -9 - (-5) = -4$

$a_4 - a_3 = -13 - (-9) = -4$

Since, the each successive term of the series has the same difference. So, it forms an AP with common difference, d = - 4

We know that, sum of n terms of an AP is

$S_n = \frac{n}{2}(2a + (n-1)d)$

Where a = first term

d = common difference

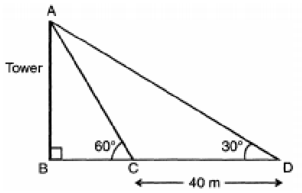
and n = no of terms

$S_{20} = \frac{20}{2}[2(-1) + (20-1)(-4)]$

= 10[- 2 - 76]

= - 780

So Sum of first 20 terms of this AP is - 780.

8. 

In $\triangle ABC$, $\tan 60^\circ = \frac{AB}{BC}$

$\Rightarrow \quad AB = \sqrt{3}BC$ .....(i)

In $\triangle ABD$,

$\tan 30^\circ = \frac{AB}{BC+40}$

$\frac{1}{\sqrt{3}} = \frac{AB}{BC+40} = \frac{\sqrt{3}BC}{BC+40}$

3BC = BC + 40

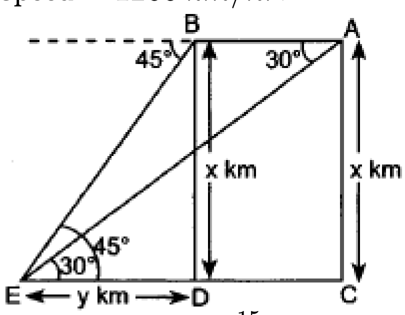
BC = 20, Hence from (i) we get

AB = 20√3 = 20 × 1.73 = 34.6 meter

OR

Distance covered in 15 seconds $= AB$

Speed $= 1200\ km/hr$.



$\therefore$ AB $= 1200 \times \frac{15}{3600} = 5\text{km}$

$AB = DC = 5km$

Let height = x km

In rt. △BDE,

$\frac{BD}{ED} = \tan 45^\circ \Rightarrow \frac{x}{y} = 1 \Rightarrow x = y$

In rt. △ACE,

$\frac{AC}{EC} = \tan 30^\circ \Rightarrow \frac{x}{y+5} = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \frac{x}{x+5} = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \sqrt{3}x = x + 5 \Rightarrow (\sqrt{3}-1)x = 5$

$\therefore \quad x = \frac{5}{\sqrt{3}-1} = \frac{5(\sqrt{3}+1)}{2} = 6.83\text{km}$

9. **Given:** PA and PB are tangents to a circle with centre O from an external point P.

To prove: OP is the right bisector of AB

Construction: Join AB. Let AB intersect OP at M.

Proof: In $\triangle MAP$ and $\triangle MBP$, we have

PA = PB [Y tangents to a circle from an external point are equal]

MP = MP [common]

△MPA = △MPB [Y tangents from an external point are equally inclined to the line segment joining the centre to that point, i.e., △ OPA = △OPB]

$\therefore \triangle MAP \cong \triangle MBP$ [by SAS-congruence].

And so, MA = MB [cpct]

and ∠AMP = ∠ BMP [cpct]

But, ∠ AMP + ∠BMP = 180° [linear pair]

$\therefore \angle AMP = \angle BMP = 90^\circ$

Hence, OP is the right bisector of AB.

10. Given equation is $(2p + 1)x^2 – (7p + 2)x + (7p – 3) = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

a = (2p + 1), b = – (7p + 2) and c = (7p – 3)

Given that the roots of equation are real and equal

Thus D = 0

Discriminant, $D = b^2 - 4ac = 0$

$[-(7p + 2)]^2 - 4.(2p + 1).(7p - 3) = 0$

Using $(a + b)^2 = a^2 + 2ab + b^2$, we get

$(49p^2 + 28p + 4) - 4(14p^2 + p - 3) = 0$

$49p^2 + 28p + 4 - 56p^2 - 4p + 12 = 0$

$-7p^2 + 24p + 16 = 0$

$7p^2 - 24p - 16 = 0$

$7p^2 - 28p + 4p - 16 = 0$

$7p(p - 4) + 4(p - 4) = 0$

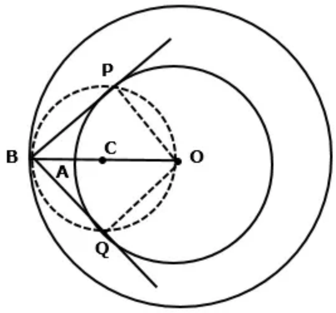
$(7p + 4)(p - 4) = 0$

$(7p + 4) = 0$ or $(p - 4) = 0$

$p = \frac{-4}{7}$ or $p = 4$

The values of p are $\frac{-4}{7}$ or 4 for which roots of the quadratic equation are real and equal.

Section C

11. 

Steps of construction:

1. Take a point O on the plane of the paper and draw a circle of radius OA = 4 cm. Also, draw a concentric circle of radius OB = 6 cm.
2. Find the midpoint C of OB and draw a circle of radius OC = BC. Suppose this circle intersects the circle of radius 4 cm at P and Q.
3. Join BP and BQ to get the desired tangents from a point B on the circle of radius 6 cm. By actual measurement, we find that BP = BQ = 4.5 cm

   Verification:

   In $\triangle$BOQ, we have OB = 6 cm and OP = 4 cm

   $\therefore OB^2 = BP^2 + OP^2$

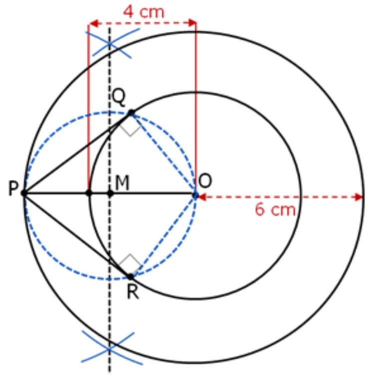
   $\Rightarrow BP = \sqrt{OB^2 - OP^2} = \sqrt{36 - 16} = \sqrt{20}$ = 4.47 cm = 4.5 cm

   Similarly, BQ = 4.47 cm = 4.5 cm.

OR

Following steps are t be taken to draw tangents on the given circle,

i. Draw a circle of 4 cm radius with centre as O on the given plane.
ii. Draw a circle of 6 cm radius takin O as its centre. Locate a point P on this circle and join OP.
iii. Bisect OP, Let M be the midpoint of PO.
iv. Taking M as its centre and MO as its radius draw a circle, let in intersects the given circle at the points Q and R.
v. Join PQ and PR. PQ and PR the required tangents.



Now we may observe that PQ and PR are 4.47 cm each. in PQO

Since PQ is tangent

$\angle \mathrm{PQO} = 90^\circ$

PO = 6cm

QO = 4 cm

Applying Pythagoras theorem in $\triangle PQO$

$\mathrm{PQ}^2 = 36 - 16$

$\mathrm{PQ}^2 = 20$

$\mathrm{PQ} = 2\sqrt{5}$

= 4.47 cm

12. The given data are:

Marks	Number of students
25 or more than 25	52
35 or more than 35	47
45 or more than 45	37
55 or more than 55	17
65 or more than 65	8
75 or more than 75	2
85 or more than 85	0

From above data we can calculate range data as following:

Marks	Number of students(f)
25 - 35	52 - 47 = 5
35 - 45	47 - 37 = 10
45 - 55	37 - 17 = 20
55 - 65	17 - 8 = 9
65 - 75	8 - 2 = 6
75 - 85	2 - 0 = 2
85 - 95	0

From table it is clear that maximum class frequency is 20 belonging to class interval 45 - 55

Modal class = 45 - 55

Lower limit (l) of modal class = 45

Class size (h) = 10

Frequency (f_1) of modal class = 20

Frequency (f_0) of class preceding modal class = 10

Frequency (f_2) of class succeeding the modal class = 9

Mode = $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$= 45 + \left(\frac{20-10}{2\times20-10-9}\right) \times 10$

$= 45 + \frac{10}{21} \times 10$

= 45 + 4.76

= 49.76

Therefore mode of data is 49.76

13. Let BC be the tower of height h metre and CD be the water tank of height h_1 metre.

In ΔABD, we have

$\tan 45^\circ = \frac{BD}{AB}$

$\Rightarrow \quad 1 = \frac{h+h_1}{40}$

$\Rightarrow \quad h + h_1 = 40\text{m}$

In ΔABC, we have

$\tan 30^\circ = \frac{BC}{AB}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{40}$

$\Rightarrow \quad h = \frac{40}{\sqrt{3}}\text{m} = \frac{40\sqrt{3}}{3}\text{m} = 23.1\text{m}$

Substituting the value of h in (i), we have

23.1 + h_1 = 40

$\Rightarrow h_1$ = (40 - 23.1) m = 16.9 m

Hence, the height of the tower is h = 23.1 m and the depth of the tank is h_1 = 16.9 m.

14. Let's first find volume of water left in sump

Volume of water left in the cuboidal samp after filling the tank = volume of cuboidal samp - volume of cylindrical tank

length(l) = 1.57 m , breadth (b) = 1.44 m

height (h) = 0.95 m

Volume of sump = l × b × h

$3.14 \times 0.6 \times 0.6 \times 0.95\text{m}^3$[Using: $V = \pi r^2 h$]

$= 3.14 \times 0.36 \times 0.95\text{m}^3$

Volume of water in the sump when it is full of water =$1.57 \times 1.44 \times 0.95\text{m}^3$

. = 2.147m^3

$\therefore$ Volume of water left in the sump after filling the tank

$= (2 \times 3.14 \times 0.36 \times 0.95 - 3.14 \times 0.36 \times 0.95)\text{m}^3$

$= 3.14 \times 0.36 \times 0.95(2-1)\text{m}^3 = 3.14 \times 0.36 \times 0.95\text{m}^3$

Area of the base of the sump = $1.57 \times 1.44\text{m}^2 = 1.57 \times 4 \times 0.36\text{m}^2 = 2 \times 3.14 \times 0.36\text{m}^2$

$\therefore$ Height of water m the sump = $\frac{3.14\times0.36\times0.95}{2\times3.14\times0.36}\text{m} = \frac{0.95}{2}$ = 0.475 = 47.5 cm

$\frac{\text{Capacity of tank}}{\text{Capacity of sump}} = \frac{3.14\times0.36\times0.95}{2\times3.14\times0.36\times0.95} = \frac{1}{2}$

Hence, the capacity of the tank is half the capacity of the sump.